

# Theory of the Plasma Resonance Probe\*

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The dc electron current to a resonance probe, as a function of frequency and other pertinent parameters, is computed from a 1-dimensional plasma-sheath model. From linearized macroscopic plasma equations with damping, the longitudinal rf field that is caused in the two-slab model by the applied rf voltage is derived. The dielectric constant in the sheath is taken to be 1; the sheath thickness is estimated from the LANGMUIR-CHILD law. In accordance with MAYER, there occurs an rf resonance at a frequency  $\omega'_{\text{res}} \lesssim \omega_s < \omega_p$ , where  $\omega_s/\omega_p$  is determined by the probe-sheath-plasma geometry. The dc electron current to the probe is computed numerically by means of an approximate classification of the (collisionless) electron orbits in the combined dc and rf fields. There follows a dc resonance at  $\omega_{\text{res}} \lesssim \omega_s$ , but none at the plasma frequency  $\omega_p$ . For  $\omega \ll \omega_{\text{res}}$  and  $\omega \gg \omega_{\text{res}}$  the correct limiting values of the dc current are reproduced. There is good qualitative agreement with recent experimental results by PETER, MÜLLER, and RABBen, but disagreement with a theory by ICHIKAWA and IKEGAMI. Arguments are presented that speak against the validity of that theory.

The plasma resonance probe, or rf probe, is a LANGMUIR probe to which, in addition to a dc voltage  $V$ , an rf voltage of amplitude  $\delta V$  and variable frequency  $\omega$  is applied<sup>1</sup>. Usually,  $V$  is taken to be negative, and the rf wave length is sufficiently large such that the rf field can be considered as purely longitudinal. One measures the dc current to the probe and/or the rf impedance of the probe.

In the following we shall consider the dc electron current only. That is, we shall ignore the frequency dependence of the dc ion current, an approximation that is legitimate at frequencies above the ion plasma frequency. Specifically, we shall not deal with the ion resonance of the dc current to the probe, an effect discovered by KATO and collaborators<sup>2</sup>.

At low frequencies the dc (electron) current is virtually independent of  $\omega$ , but larger than the static current:<sup>1</sup>

$$j_e \rightarrow j_Q = j_S I_0(\delta\eta), \quad \omega \rightarrow 0, \quad (1)$$

with the static current density

$$j_S = n_0 e (kT_e / 2 \pi m_e)^{1/2} \exp(-\eta). \quad (2)$$

Here  $\eta = -eV/kT_e$ ,  $\delta\eta = e\delta V/kT_e$ ,  $I_0$  = modified BESSEL function of zeroth order. The quantity  $j_Q$  may be called the "quasistatic current density". At a dc resonance frequency  $\omega_{\text{res}} < \omega_p$  ( $\omega_p$  = electron plasma frequency) the dc current goes through a resonance peak and then, for  $\omega \gg \omega_{\text{res}}$ , approaches the static value  $j_S$ . Sometimes secondary peaks are seen.

Experimental investigations of the resonance probe have been carried out by several authors<sup>1-10</sup>. While in some of the investigations<sup>1, 6-8</sup> the dc resonance frequency was claimed to coincide with the plasma frequency, other investigators<sup>3-4, 10</sup> showed that this is not, in general, true. Rather the dc resonance frequency  $\omega_{\text{res}}$  and the rf resonance frequency  $\omega'_{\text{res}}$  appeared to satisfy relations of the form

$$\omega_{\text{res}} \approx \omega'_{\text{res}} \approx \omega_s \equiv \omega_p \sqrt{s/q} < \omega_p, \quad (3)$$

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<sup>1</sup> K. TAKAYAMA, H. IKEGAMI, and S. MIYAZAKI, Phys. Rev. Letters **5**, 238 [1960].

<sup>2</sup> K. KATO, K. OGAWA, M. YOSELI, and H. SHIMAHARA, J. Phys. Soc., Japan **19**, 1849 [1963].

<sup>3</sup> G. PETER, G. MÜLLER, and H. H. RABBen, VI. Int. Conf. on Ionization Phenomena in Gases, Paris 1963. — Lab. Report IPP/2/32, Institut für Plasmaphysik, Garching bei München 1963.

<sup>4</sup> G. V. GIERKE, G. MÜLLER, G. PETER, and H. H. RABBen, Z. Naturforsch. **19 a**, 1107 [1964].

<sup>5</sup> H. M. MAYER, VI. Int. Conf. on Ionization Phenomena in Gases, Paris 1963.

<sup>6</sup> R. B. CAIRNS, Proc. Phys. Soc., Lond. **82**, 243 [1963].

<sup>7</sup> H. IKEGAMI, K. TAKAYAMA, S. HAGIWARA, and S. KOJIMA, VI. Int. Conf. on Ionization Phenomena in Gases, Paris 1963.

<sup>8</sup> H. IKEGAMI and K. TAKAYAMA, Lab. Report IPPJ-10, Institute of Plasma Physics, Nagoya Univ., Nagoya, Japan 1963.

<sup>9</sup> J. URAMOTO, H. IKEGAMI, and K. TAKAYAMA, Lab. Report IPPJ-15, ibid. 1963.

<sup>10</sup> J. URAMOTO, J. FUJITA, H. IKEGAMI, and K. TAKAYAMA, Lab. Report IPPJ-19, ibid. 1963.



where the ion sheath thickness  $s$  may be approximated by the LANGMUIR-CHILD formula in the case of a plane sheath:

$$s = 1.25 r_D \eta^{3/4} (T_e/T_i)^{1/4}, \quad (4)$$

$$r_D = (kT_e/4\pi n_0 e^2)^{1/2}, \quad (4a)$$

and where  $q$  is a characteristic length discussed below, which depends on the probe-sheath-plasma geometry. Our results presented below will show that, for  $s \ll q$ , Eq. (3) is to be replaced by

$$\omega_{\text{res}} \text{ and } \omega'_{\text{res}} \lesssim \omega_s. \quad (3a)$$

For  $s \ll q$ ,  $q$  depends little on  $s$ . Hence, at constant plasma density and temperature,  $\omega_s$  increases with increasing absolute magnitude of the negative dc voltage, and at constant plasma temperature and dc voltage,  $\omega_s$  increases less strongly with increasing carrier density  $n_0$  than does  $\omega_p$ . Both effects have been verified experimentally<sup>3, 4</sup>. As  $q$  depends on the size and the shape of the probe, so does  $\omega_s$ .

A formula similar to Eq. (3) was derived by MAYER<sup>5</sup> for the (approximate) rf resonance frequency  $\omega_s$  of a plane plasma condenser. The model used was a three-slab dielectric of the form sheath-plasma sheath, with the dielectric constants of the sheaths equated to 1. The dielectric constant of the plasma (= electron fluid) was taken to be

$$\epsilon_p = 1 - \omega_p^2/(\omega^2 + i\nu\omega), \quad (5)$$

relative to the FOURIER mode  $\exp(-i\omega t)$ , where  $\nu$  is a collisional damping frequency. A formula analogous to Eq. (3) was also derived by VANDENPLAS and GOULD<sup>11</sup> in a different connection. HARP<sup>12</sup> used a model similar to MAYER's in the theoretical study of the rf resonance of a spherical probe. OTT<sup>13</sup> derived expressions for  $\omega_s$  for various probe geometries, and for a plane resonance probe in a magnetic field.

A general formula for the approximate rf resonance frequency  $\omega_s$  is easily derived along the same lines, if no magnetic field is present. It comprises the above-mentioned formulae as special cases:

$$\omega_s = \omega_p \sqrt{1 - C/C'} < \omega_p. \quad (6)$$

In the case of a double probe (stray-field neglected)  $C$  is the vacuum capacity of the double probe, while  $C'$  is the vacuum capacity of the system formed of the two plasma-sheath boundaries. The two sheaths are assumed not to overlap. By deforming one of the components of the double probe, together with its plasma-sheath boundary, into a sphere of infinite radius, one obtains  $\omega_s$  for a single probe that is situated far from other electrodes. Equation (6) is an approximation in that it is derived with the assumption that the plasma-sheath boundaries are equipotential surfaces in the corresponding pure vacuum problem (i. e., same electrodes present, but plasma absent). This assumption is exactly valid for probes of spherical, cylindrical, or plane symmetry, and approximately valid, if the sheath thicknesses  $s_1, s_2$  are sufficiently small, such that the sheaths may be considered as plane. In the latter case we have:

$$\omega_s \approx \omega_p \sqrt{4\pi C(s_1/F_1 + s_2/F_2)} \quad (7)$$

for the double probe, where  $F_1, F_2$  are the areas of the probe surfaces, and for the single probe:

$$\omega_s \approx \omega_p \sqrt{4\pi C s/F}. \quad (7a)$$

Comparison with Eq. (3) shows the exact meaning of the quantity  $q$ .

Equation (6) gives a possible explanation for the fact that several authors<sup>1, 6-8</sup> have found  $\omega_{\text{res}}/\omega_p$  to be independent of the dc voltage  $\eta$  and the plasma density  $n_0$ . At low plasma density and for small probe radius the sheath thickness may become greater than the probe radius, especially for large  $\eta$ . Then, in Eq. (6),  $C \ll C'$ , and  $\omega_s \approx \omega_p$ .

From the above-mentioned investigations<sup>3-5, 11-12</sup> and from related ones<sup>14-17</sup> the conclusion can be drawn that the rf resonance and the dc resonance are related phenomena (the resonance frequencies are nearly the same) and that the plasma-sheath inhomogeneity is essential in both effects.

The above-mentioned theoretical work<sup>5, 11-13</sup> concerns only the (longitudinal) rf resonance. A theory

<sup>11</sup> P. E. M. VANDENPLAS and R. W. GOULD, Proc. V. Int. Conf. on Ionization Phenomena in Gases (Munich 1961), North Holland Publ. Co., Amsterdam 1962, Vol. 2, p. 1470.

<sup>12</sup> R. S. HARP, A Theory of the Resonance Probe, Microwave Lab., W. W. Hansen Labs. of Physics, Stanford Univ., Stanford, Calif. 1963.

<sup>13</sup> W. OTT, Internal Memorandum, Institut für Plasmaphysik, Garching bei München 1964.

<sup>14</sup> R. HARP and G. S. KINO, M. L. Report No. 1076, Microwave Lab., W. W. Hansen Labs. of Physics, Stanford Univ., Stanford, Calif. 1963.

<sup>15</sup> J. PAVKOVICH and G. S. KINO, M. L. Report No. 1077, *ibid.* 1963.

<sup>16</sup> R. S. HARP, G. S. KINO, and J. PAVKOVICH, Phys. Rev. Letters **11**, 310 [1963].

<sup>17</sup> S. M. LEVITSKII and I. P. SHASHURIN, Soviet Phys.—Tech. Phys. **8**, 319 [1963].

of the *dc resonances* has not been given, except for the work by ICHIKAWA and IKEGAMI, and ICHIKAWA<sup>18-20</sup>. However, this theory suffers from the neglect of the above-mentioned longitudinal rf resonance at  $\omega'_{\text{res}} < \omega_p$ . Furthermore these authors normalize the longitudinal rf field contrary to the basic concepts of electrostatics in that the path integral not of the total rf field (=sum of external and internal fields), but of the external field only, is equated to the negative rf voltage. This improper normalization leads to an unphysical rf resonance at  $\omega = \omega_p$ . Also the distinction between the internal rf field, that is produced by the charge density of the plasma, and the external rf field, whose sources are external charges, is not clear-cut. A well-behaved external field should not be affected by the plasma, but the above authors assume it to be shielded out within a distance of the order of the ion sheath thickness. On the other hand, the external field is supposed to influence the homogeneous plasma, i. e. to modify the velocity and density distribution and to induce an internal rf field. It is this latter field which then is disregarded in the normalization. In the end ICHIKAWA and IKEGAMI obtain a dc resonance frequency  $\omega_{\text{res}} \approx \omega_p$  independent of the probe-sheath-plasma geometry. As mentioned, this contradicts experimental results. Furthermore, in his improved theory that takes into account the nonvanishing transit time across the sheath, ICHIKAWA<sup>20</sup> finds a minimum of the dc current to occur at  $\omega > \omega_{\text{res}}$ . The value of this current minimum is smaller than the static current. Such a minimum has never been observed in experiments.

In the present paper an alternative theory of the dc current to the resonance probe is given that takes the rf resonance produced by the plasma-sheath inhomogeneity into account. A simple one-dimensional discontinuous plasma-sheath model, similar to MAYER's<sup>5</sup>, will be used. More details of the work than are given below are to be found in a Laboratory Report by the author<sup>21</sup>.

## I. Theory

The computation of the dc electron current to the rf probe will be done for negative dc voltage  $V$ . The following one-dimensional model is used in the deri-

vation of the rf field (Fig. 1). Adjacent to the probe there is the dc ion sheath of thickness  $\delta$ , where  $\delta$  is given by Eq. (4). In  $0 < x < R - \delta$  there is a perturbed plasma region, while for  $x < 0$  the plasma is assumed unperturbed. The dielectric constant  $\epsilon_s$  in the sheath is equated to 1, while in the plasma Eq. (5) is used. The characteristic length  $R$  is chosen such that the rf resonance frequency  $\omega_s$  obtained from this model obeys Eq. (7 a), with  $s = \delta$ . Hence  $R = (F/4\pi C)$ . This one-dimensional model is most appropriate for a plane resonance probe; for a plane probe of circular shape with radius  $r$ ,  $R = \pi r/4$ . On the basis of this simple model the derivation of the rf field is straight-forward and need not be given here. The resulting expressions for the rf field are listed in Eqs. (12) and (20) to (24).

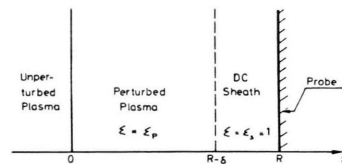


Fig. 1. One-dimensional plasma-sheath model of the resonance probe.

The plasma dielectric constant  $\epsilon_p$  of Eq. (5) is an approximate expression. It may be derived from the macroscopic equations of motion of the electrons (with collisional damping) and from the Poisson equation by letting the wave-number  $k \rightarrow 0$ . By this device, rf screening within the *homogeneous* plasma ( $\omega < \omega_p$ ) and running waves ( $\omega > \omega_p$ ) are dropped. This is easily justified for  $\omega \ll \omega_p$ , in which case the rf screening should occur within the dc sheath, and for  $\omega \gg \omega_p$ , in which case the running wave terms have negligible amplitudes. For  $\omega \sim \omega_p$  calculations that take the  $k$ -dependence into account approximately, have shown that neglecting the  $k$ -dependence has no serious effect upon the theoretical result for the dc current to the probe<sup>21</sup>.

The dc electron current density  $j$  to the probe will be computed by means of an approximate classification of the collisionless electron orbits in the combined dc and rf fields:

$$j = j_0 (2\pi)^{-1} \int_0^{2\pi} d\varphi \int_0^\infty d\beta_0 e^{-\beta_0} \text{He}(\beta_R), \quad (8)$$

<sup>18</sup> Y. H. ICHIKAWA and H. IKEGAMI, Progr. Theor. Phys. **28**, 315 [1962].

<sup>19</sup> Y. H. ICHIKAWA, VI. Int. Conf. on Ionization Phenomena in Gases, Paris 1963. — C. R. Acad. Sci., Paris **256**, 3434 [1963].

<sup>20</sup> Y. H. ICHIKAWA, Research Reports NUP-63-12, NUP-63-14, NUP-63-15, Dept. of Physics, College of Science and Engineering, Nihon Univ., Tokyo, Japan 1963.

<sup>21</sup> H. K. WIMMEL, Lab. Rep. IPP/6/11, Institut für Plasma-physik, Garching bei München 1963.

where the unperturbed velocity distribution  $f_0(v_0)$  of the electrons has been assumed to be Maxwellian. Here  $j_0$  is the thermal current density of electrons:

$$j_0 = n_0 e (k T_e / 2 \pi m_e)^{1/2}, \quad (9)$$

$\varphi = \omega t_0$  is the rf phase at  $t_0$ , the time at which an electron passes through  $x=0$  from left to right,  $\beta_0$  and  $\beta_R$  are its kinetic energies in units of  $k T_e$  at  $x=0$  and  $x=R$ , and  $\text{He}(x)$  is the *Heaviside* step function, i. e.,

$$\text{He}(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 & \text{for } x > 0 \end{cases}.$$

The approximation in Eq. (8) consists in assuming that all those and only those electrons with  $\beta_R > 0$  will actually reach the probe. This is a good approximation for  $\delta V$  and  $\omega$  sufficiently small, and for particle speeds sufficiently large, i. e. if the total field an electron sees during his passage to the probe is sufficiently close to being a monotonic function of  $x(t)$ . From conservation of energy there follows, if collisions are neglected,

$$\beta_R = \beta_0 - \eta + \frac{e}{k T_e} \int_0^R E_w[x, t(t_0, x)] dx, \quad (10)$$

where now  $\eta = +eV/kT_e$ ,  $E_w$  the rf field, and  $t$  = time. Here and in the following the electron charge is put equal to  $(+e)$ , and  $V > 0$ .

For simplicity we determine  $t$  as if the electron speeds were constant in time, i. e., we put

$$t = t_0 + x/v_0. \quad (11)$$

In the perturbed plasma region this is a good approximation for most electrons, if the rf field is sufficiently small. In the sheath it is justified for those electrons for which the time of flight through the sheath is not longer than half the rf period. An estimate using the LANGMUIR-CHILD law for the sheath thickness  $\delta$  shows that for an electron with critical  $v_0$ , i. e., such that  $\beta_0 = \eta$ , the time of flight is sufficiently short, if  $\omega \leq \omega_p$  and  $\eta < 10$ . For electrons so slow that their transit time amounts to many half-periods, it is expected that the errors made for electrons of comparable  $v_0$  will cancel out. In an intermediate  $v_0$ -range, use of Eq. (11) may lead to a certain systematic error in the result.

In accordance with the above-described model, the rf field has the form:

$$E_w(x, t) = \frac{\delta V}{R} [X_m \cos(\omega t) + Y_m \sin(\omega t)] \quad \text{for } R_m \leq x < R_{m+1}, \quad m = 1, 2. \quad (12)$$

The values of the rf field coefficients  $X_m$ ,  $Y_m$  and of the distances  $R_m$  are given at the end of this Section.

On using Eq. (12), the  $x$ - and  $\varphi$ -integrations can be carried out, and the result for the dc current is, by straight-forward analysis:

$$j = j_s \left\{ 1 + \int_0^\infty d\beta_0 e^{-a} \text{sgn}(-a) G\left(\left|\frac{a}{b}\right|\right) \right\}, \quad (13)$$

with

$$G(x) = \begin{cases} \pi^{-1} \arccos x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } 1 < x < \infty \end{cases}, \quad (14)$$

$$a = \beta_0 - \eta, \quad b = (b_1^2 + b_2^2)^{1/2}, \quad (15), \quad (16)$$

$$b_1 = (\delta\eta/\chi_3) \sum_{n=1}^2 [X_n(\sin \chi_{n+1} - \sin \chi_n) - Y_n(\cos \chi_{n+1} - \cos \chi_n)], \quad (17)$$

$$b_2 = -(\delta\eta/\chi_3) \sum_{n=1}^2 [Y_n(\sin \chi_{n+1} - \sin \chi_n) + X_n(\cos \chi_{n+1} - \cos \chi_n)], \quad (18)$$

where

$$\chi_n = (R_n \omega / r_D \omega_p) (2\beta_0)^{-1/2}, \quad (19)$$

and  $R_1 = 0$ ;  $R_2 = R - \delta$ ;  $R_3 = R$  (Fig. 1).

With an rf voltage of the form  $U_w = \delta V \cos(\omega t)$  applied between  $x=0$  and  $x=R$  (Fig. 1), the field coefficients  $X_n$ ,  $Y_n$  are given by the following equations:

$$X_1 = -\frac{\omega^2(\omega^2 - \omega_s^2 + \nu^2)}{(\omega^2 - \omega_s^2)^2 + (\nu\omega)^2}, \quad (20)$$

$$Y_1 = -\frac{\nu\omega\omega_s^2}{(\omega^2 - \omega_s^2)^2 + (\nu\omega)^2}, \quad (21)$$

$$X_2 = -\frac{(\omega^2 - \omega_p^2)(\omega^2 - \omega_s^2) + (\nu\omega)^2}{(\omega^2 - \omega_s^2)^2 + (\nu\omega)^2}, \quad (22)$$

$$Y_2 = -\frac{\nu\omega(\omega_p^2 - \omega_s^2)}{(\omega^2 - \omega_s^2)^2 + (\nu\omega)^2}, \quad (23)$$

with  $\nu$  denoting the damping frequency of Eq. (5), and

$$\omega_s = \omega_p \sqrt{\delta/R}. \quad (24)$$

Equations (13) to (24) were evaluated numerically on an electronic computer (IBM 7090).

## II. Results

That the rf field given by Eqs. (12) and (20) to (23) shows a resonance for  $\omega'_{\text{res}} \lesssim \omega_s$  is demonstrated in Figs. 2 and 3 for two special cases, i. e., for  $\nu/\omega_p = 0.6$ , and  $\delta/R = \omega_s^2/\omega_p^2 = 0.3$  and  $0.01$ .

Shown are the amplitudes  $U_p$  and  $U_s$  and the phases  $\varphi_p$ ,  $\varphi_s$  of the rf component voltages in the plasma and in the sheath, as functions of  $10 \log(\omega/\omega_p)$ . The total rf amplitude  $\delta V$  has been normalized to 1. The rf resonance is indicated by  $(U_p + U_s)$  becoming greater than 1. Similar results obtain for different values of  $\omega_s$ . The discrepancy between  $\omega'_{\text{res}}$  and  $\omega_s$  for  $\delta \ll R$  is to be attributed to the nonvanishing damping parameter  $\nu$ .

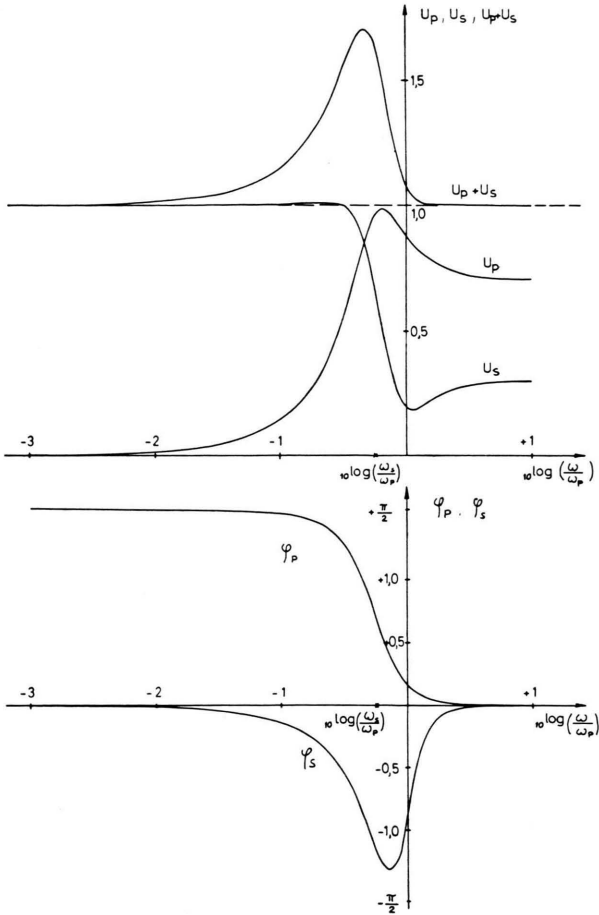


Fig. 2. Amplitudes  $U$  and phases  $\varphi$  of the rf component voltages in the plasma (p) and in the sheath (s) as functions of  $10 \log(\omega/\omega_p)$ . Theoretical curves for  $\delta/R = \omega_s^2/\omega_p^2 = 0.3$  and  $\nu/\omega_p = 0.6$ . Normalization:  $\delta V = 1$ .

Figs. 4 and 5 show the dc electron current density to the resonance probe, in units of the static current density, as a function of  $10 \log(\omega/\omega_p)$ , for  $T_i = T_e$ ,  $r_D/R = 0.1$  and  $0.01$ ,  $\nu/\omega_p = 0.6$ , and for several values of  $\eta$  and  $\delta\eta$ . The asymptotic values of the current for  $\omega \rightarrow 0$  and  $\omega \rightarrow \infty$  may be derived by separate considerations; the computed curves agree

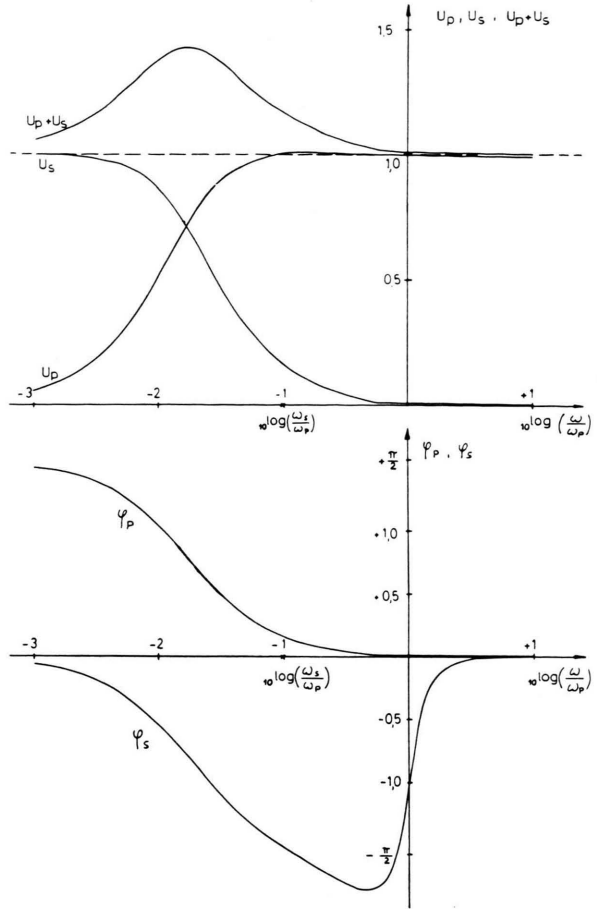


Fig. 3. Amplitudes and phases of the rf component voltages in the plasma and in the sheath for  $\delta/R = \omega_s^2/\omega_p^2 = 0.01$  and  $\nu/\omega_p = 0.6$ .

very well with these asymptotic values. The dc resonance occurs at  $\omega_{\text{res}} < \omega_p$ , at  $\omega = \omega_p$  there is no resonance. The small maxima at  $\omega > \omega_{\text{res}}$  would probably be less distinct, or absent, if a continuous plasma-sheath model had been used; in the experimental results they do not show up. On the other hand, experiments<sup>3</sup> have occasionally shown secondary resonance peaks of the dc current at  $\omega = \omega_{\text{res}}/2$ ,  $\omega_{\text{res}}/3$ ,  $\omega_{\text{res}}/4$ ; these are not described by the present theory, but would show up as an effect of higher harmonics, if the rf field were calculated by a non-linear theory.

The model used leaves out of consideration trapped electrons or ones repeatedly reflected by the electric field, before they eventually either reach the probe or leave the perturbed plasma region. Accordingly, the rf amplitudes  $\delta\eta$  were chosen such that slow elec-



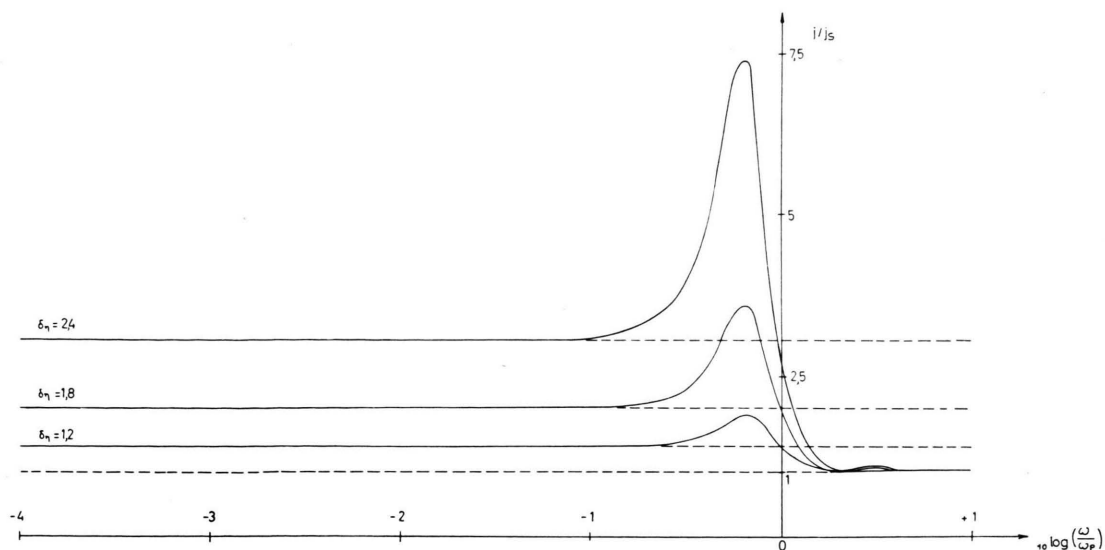


Fig. 4. Dc electron current to the resonance probe in units of the static current versus  $10 \log(\omega/\omega_p)$  for  $r_D/R=0.1$ ,  $\nu/\omega_p=0.6$ ,  $\eta=6$ . Dotted lines: static current and quasistatic limits.

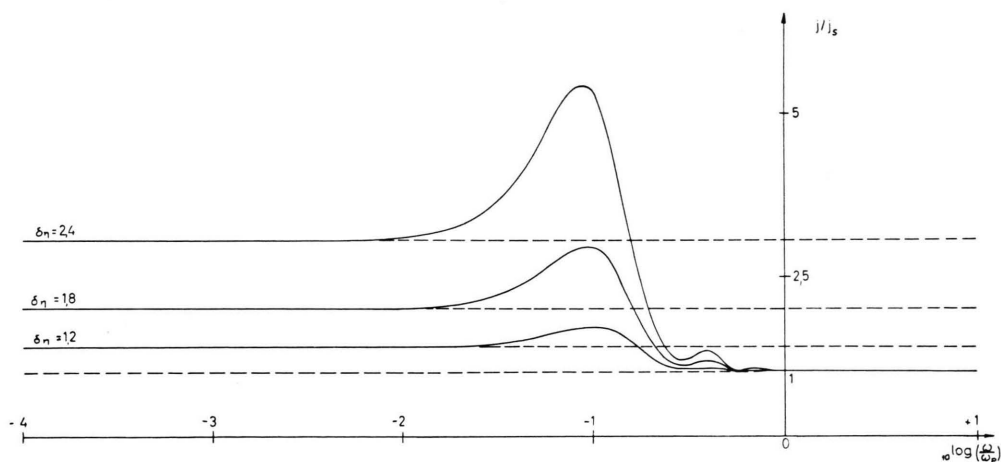


Fig. 5. Dc electron current in units of the static current versus  $10 \log(\omega/\omega_p)$  for  $r_D/R=0.01$ ,  $\nu/\omega_p=0.6$ ,  $\eta=9$ .

trons, with  $\beta_0 \ll 1$ , should not contribute to the dc current increment over and above the static current.

The dc electron current density was evaluated also for several other values of the parameters  $\nu/\omega_p$  and  $\eta$ . We do not list the results, but mention only some characteristic variations in the results. As  $r_D/R$  is decreased from 0.1 to 0.01, with the other parameters kept constant,  $\omega_{\text{res}}$  decreases, so do the halfwidths, while the peak currents keep their order of magnitude, but change their  $\eta$ -dependence. As  $\nu/\omega_p$  is decreased from 0.6 to 0.2,  $\omega_{\text{res}}$  increases, though still  $\omega_{\text{res}} < \omega_s$ ; the halfwidths of the resonance peaks decrease, and the peak currents increase.

As  $\eta$  is increased from 3 to 12, so is the dc resonance frequency  $\omega_{\text{res}}$ , together with the parameter  $\omega_s$ . Fig. 6 shows the quantities  $\omega_{\text{res}}$  and  $\omega_s$  as functions of  $\eta$  for  $\nu/\omega_p=0.6$ ,  $r_D/R=0.1$  and 0.01, and for several values of  $\delta\eta$ . The dc resonance frequency is seen to depend but little on  $\delta\eta$ , as is the case in experiments. Also  $\omega_{\text{res}} < \omega_s$ , cf. Eq. (24), but  $\omega_{\text{res}}/\omega_s$  is nearly independent of  $\eta$ . This agrees well with experimental results<sup>4</sup>, in which good agreement is found between the dependence on the dc voltage of the experimental  $\omega_{\text{res}}$  and that of a theoretical  $\omega_s$  that is derived on the basis of the LANGMUIR-CHILD law. Even better agreement between  $\omega_{\text{res}}$  and  $\omega_s$  is

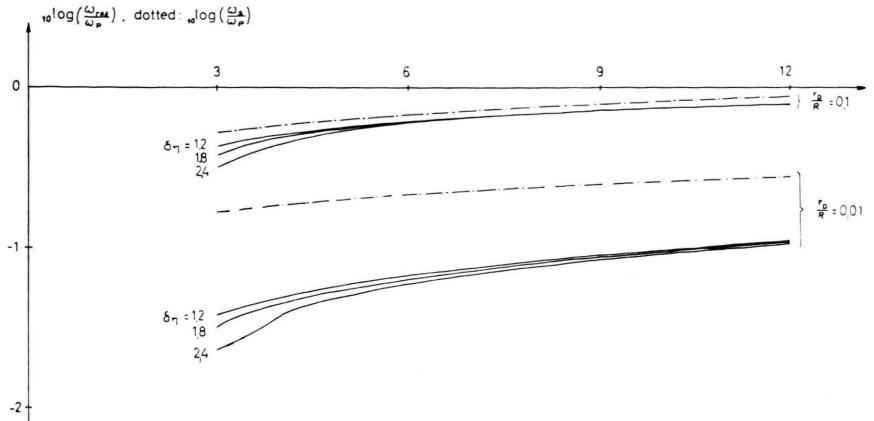


Fig. 6. Dc resonance frequency  $\omega_{\text{res}}$  (solid lines) and parameter  $\omega_s$  (dotted) versus  $\eta$  for  $\nu/\omega_p = 0.6$ ,  $r_D/R = 0.1$  and  $0.01$ .

obtained for  $\nu/\omega_p = 0.2$ ; yet in this case the dc current resonance peaks are markedly higher than the ones obtained for  $\nu/\omega_p = 0.6$ .

### III. Discussion

The results obtained for the dc electron current to the resonance probe allow a simple interpretation (compare Figs. 1 to 3).

1. For  $\omega \ll \omega'_{\text{res}}$  the rf voltage is localized in the dc sheath. During the transit time of an electron through the sheath the total probe voltage remains virtually unaltered. Hence, the instantaneous probe current can be calculated as in the time-independent case by the LANGMUIR theory. By forming the time average the result of Eq. (1) is obtained.

2. For  $\omega \approx \omega'_{\text{res}}$  there occurs a resonance increase of the rf fields in the plasma and the sheath such that the sum of the component voltage amplitudes in the plasma and in the sheath — both are now of comparable magnitude — becomes larger than the total rf voltage amplitude. On the basis of the model used in the calculation, it is easy to see that this rf resonance is a necessary condition for the dc resonance. At rf resonance the electrons may “see” an effective rf voltage amplitude greater than the applied rf amplitude, by virtue of their non-vanishing transit times from the unperturbed plasma to the probe. Even though, for a single electron, the probabilities for an enlarged additional gain and an enlarged additional loss of kinetic energy are nearly equal, the MAXWELL distribution of the initial velocities  $v_0$  has the consequence that a resonance increase of the dc current, over and above the quasi-

static limit, comes about as a difference effect. As the transit times of the electrons are important — the dc resonance vanishes for vanishing transit times —, it is plausible that  $\omega_{\text{res}}$  and  $\omega'_{\text{res}}$  do not coincide exactly.

3. For  $\omega \gg \omega'_{\text{res}}$ , again an appreciable part, in some cases: the dominant part, of the rf voltage drop extends through the perturbed plasma. There is no more rf resonance, i.e., the sum of the component amplitudes in the plasma and in the sheath equals the total rf voltage amplitude. The transit times of the electrons across the perturbed plasma and the sheath now amount to several periods of the rf field. Hence, the effective rf voltage amplitude “seen” by the electrons becomes small compared to the applied rf amplitude, and consequently the dc current increment over and above the static current goes to zero, as  $\omega$  is increased.

Although the above theoretical results correspond reasonably well with the experimental findings, the model used in the calculation shows some weaknesses. It turns out that the collisional damping frequency  $\nu$  must be of the order of  $\omega_p$  in order that realistic results may be obtained. Since in some experiments<sup>3, 4</sup> the electron mean free path is larger than the diameter of the plasma container, it is clear that under such circumstances  $\nu$  describes not damping by collisions, but some other damping process, perhaps LANDAU damping or a damping effect caused by details of the sheath structure. Specifically it was suggested by MAYER<sup>5</sup> and discussed in more detail by GOULD<sup>22</sup> that stochastic acceleration, or inelastic

<sup>22</sup> R. W. GOULD, Internal Memorandum, Institut für Plasma-physik, Garching bei München 1964.

reflection, of the plasma electrons by the oscillating sheath would provide for a damping of the right order of magnitude. This possibility was not studied in the present work. Another problem is presented by the discontinuity of the plasma-sheath model used. It exhibits but one rf resonance frequency in a linear theory, while a plasma-sheath system with continuous density variation ought to exhibit several such resonance frequencies<sup>23, 24</sup>. However, it is plausible that such additional resonances would show up mainly in rf measurements, and not so much in the dc current to the resonance probe, since the spatial structures of the associated fields are more complex<sup>25</sup>.

#### IV. Conclusion

It is concluded that the above one-dimensional discontinuous plasma-sheath model yields results for the dc electron current to the resonance probe that correspond reasonably well with experimental results. It follows that the influence of the plasma-sheath inhomogeneity upon the rf field is essential for the occurrence of the resonance. There are also the following practical consequences concerning the use of the resonance probe as a diagnostic tool. The elec-

tron plasma frequency cannot be determined from the measurement of  $\omega_{\text{res}}$  alone. Possibly a sufficiently large variation of  $\eta$  or the combined use of several probes of different sizes will permit to determine  $\omega_p$ . For  $\omega_s \ll \omega_p$  the discrepancy between  $\omega_{\text{res}}$  and  $\omega_s$ , resp.  $\omega'_{\text{res}}$  and  $\omega_s$ , is to be remembered. Furthermore, a determination of the actual electron collision frequency from resonance probe measurements turns out to be impractical, whenever the collision frequency is small compared to the plasma frequency. This follows from the fact that the parameter  $\nu$  must be chosen of the order of  $\omega_p$  in order to obtain agreement with experiments in which collisions are negligible. On the other hand, if the pertinent plasma parameters are known, the resonance probe may be used in investigations of the ion sheath structure.

Concerning the theory by ICHIKAWA and Ikegami<sup>18-20</sup> it has been shown that it does not describe the experimental results, and that several theoretical arguments against its validity may be raised.

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Stimulating conversations with Professor Dr. A. SCHLÜTER, Dr. G. MÜLLER, and Mr. H. VÖLK, and advice and assistance with computer programming by Mrs. G. HAIN are gratefully acknowledged.

<sup>23</sup> J. C. NICKEL, J. V. PARKER, and R. W. GOULD, *Phys. Rev. Letters* **11**, 183 [1963].

<sup>24</sup> J. V. PARKER, J. C. NICKEL, and R. W. GOULD, to be published in *Phys. Fluids*.

<sup>25</sup> R. W. GOULD, private communication.